$$- \quad + \quad r \quad +$$
$$| — p_1 — | — p_2 — | — p_3 — |$$
$$+ \quad - \quad s \quad -$$

$1-p_1-p_2+p_1p_2$

0 $\left\{\begin{array}{cccc} - & + & r & + \\ + & - & s & - \end{array}\right.$

1 $\left\{\begin{array}{cccc} - & - & s & - \\ + & + & r & + \end{array}\right.$ $\quad p_1 q_2 q_3 \quad 72 \quad 0.1017 \quad 72$

2 $\left\{\begin{array}{cccc} - & + & s & - \\ + & - & r & + \end{array}\right.$ $\quad q_1 p_2 q_3 \quad 130 \quad 0.1838 \quad 130$

3 $\left\{\begin{array}{cccc} - & + & r & - \\ + & - & s & + \end{array}\right.$ $\quad q_1 q_2 p_3 \quad 71 \quad 0.1003 \quad 71$

12 $\left\{\begin{array}{cccc} - & - & r & + \\ + & + & s & - \end{array}\right.$

13 $\left\{\begin{array}{cccc} - & + & s & + \\ + & + & r & - \end{array}\right.$

23 $\left\{\begin{array}{cccc} - & + & s & + \\ + & - & r & - \end{array}\right.$

123 $\left\{\begin{array}{cccc} - & - & r & - \\ + & + & s & + \end{array}\right.$ $\quad p_1 p_2 p_3 \quad 5 \quad 0.0071 \quad 5$

$$\frac{}{278} \qquad \frac{0.0071}{0.3929}$$

707.6

$p_1 - p_1p_2 - p_1p_3 + p_1p_2p_3$

$p_2 - p_1p_2 - p_2p_3 + \cdots$

$p_3 - p_1p_3 - p_2p_3 + \cdots$

$- - -$

$p_1+p_2+p_3 - 2p_1p_2 - 2p_1p_3 - 2p_2p_3 + 4p_1p_2p_3$

$\cancel{p_1(1-p_2p_3+p_2p_3p_2)}$

$$\frac{p_2p_3}{q_2q_3} = \frac{5}{72} = 0.06944$$

$$\frac{p_1p_3}{q_1q_3} = \frac{5}{130} = 0.03846$$

$$\frac{p_1p_2}{q_1q_2} = \frac{5}{71} = 0.07042.$$

$A = \frac{p_1}{q_1} \quad B = \frac{p_2}{q_2} \quad C = \frac{p_3}{q_3}$

$\frac{p}{1-p} = x \qquad x - xp = p \qquad x = p(1+x) \qquad p = \frac{x}{1+x}$

$AB = x = 0.07042 \quad A = \frac{x}{B}$

$AC = y = 0.03846 \quad \frac{Cx}{B} = y \quad \frac{C}{B} = \frac{y}{x}$

$BC = z$ [illegible]

$p \quad q$

$p_1 = \frac{0.1975}{1.1975} = 0.1649 \quad 0.8351$ [illegible]

[illegible]

TABLE 38/70+

The I.Q. and I.B. scores of 65 ~~Bristol~~ Bath children (Robert's and Griffiths)

I.Q. (x)	I.B. (y)	I.Q. (x)	I.B. (y)		I.Q. (x)	I.B. (y)
67	36	91	91		108	91
70	28	91	129		108	111
72	34	92	92		108	115
74	28	92	98		109	134
75	48	94	115		110	113
76	50	95	80		110	124
77	62	96	96		110	129
78	22	96	108		110	140
81	82	96	146		112	145
82	84	97	118		113	147
83	64	97	121		114	126
83	77	99	106		114	132
83	82	100	79		115	142
84	92	101	103		115	157
85	91	101	113		116	126
86	65	101	118		116	138
86	75	101	119		123	149
86	76	101	141		126	142
87	68	103	115		126	164
89	80	103	131		127	172
89	110	103	139		135	156
91	72	107	102	Total	6366	6739

TABLE ~~10.2~~ 39

The Combination of Correlation Coefficients (Roberts and Griffiths)

Group of Children	r	Z	n	I_z (= n - 3)	$I_z Z$
1	0.8859	1.4026	65	62	82.9612
2	0.9257	1.6274	60	57	92.7618
3	0.8749	1.3537	67	64	86.6368
			Total	183	266.3598

$$\hat{Z} = \frac{S(I_z Z)}{S(I_z)} = 1.4555$$

$$\hat{r} = 0.8968$$

$$V_{\hat{Z}} = \frac{1}{S(I_z)} = \frac{1}{183}$$

A B C ~~A~~ B_1 B [illegible]

+	−	+
−	+	−

$0 \begin{cases} \cancel{+\ -\ +} \\ -\ +\ - \end{cases}$ $(1-p_1)(1-p_2)$

3

$1 \begin{cases} +\ +\ - \\ \cancel{-\ -\ +} \end{cases}$ $p_1(1-p_2)$

10

$r = \frac{p_2}{1-p_2}$

$r =$

$2 \begin{cases} \cancel{+\ -\ -} \\ -\ +\ + \end{cases}$ $p_2(1-p_1)$

$r = \frac{p_2}{1-p_2}$

1

$1{\cdot}2 \begin{cases} +\ +\ + \\ \cancel{-\ -\ -} \end{cases}$ $p_1 p_2$

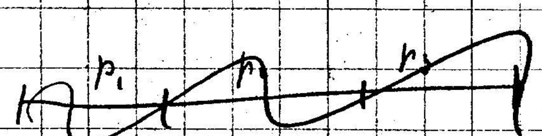
30
3
10
1

$p_1 = \frac{1}{11}$ $= 9\%$

$p_2 = \frac{1}{4}$ $= 25\%$ needing adjustment.

150 59
51 18
201 77

31



273

1②

Hence $$\frac{1}{2}\frac{\partial T}{\partial \lambda_1} = \frac{T}{D}\frac{\partial D}{\partial \lambda_1}$$

and $$\frac{1}{2}\frac{\partial T}{\partial \lambda_2} = \frac{T}{D}\frac{\partial D}{\partial \lambda_2}$$

$\frac{T}{D}$ Since T/D is ~~a constant occurring in~~ common to both equations it may be removed to give two new equations whose solutions will themselves be proportional to the solutions of the two equations obtained by the maximisation process.

3½

Now $T = \lambda_1^2 S(x_1-\bar{x}_1)^2 + 2\lambda_1\lambda_2 S[(x_1-\bar{x}_1)(x_2-\bar{x}_2)] + \lambda_2^2 S(x_2-\bar{x}_2)^2$

and $D = \lambda_1 d_1 + \lambda_2 d_2$

(1½)

So $\frac{\partial T}{\partial \lambda_1} = 2\lambda_1 S(x_1-\bar{x}_1)^2 + 2\lambda_2 S[(x_1-\bar{x}_1)(x_2-\bar{x}_2)]$, $\frac{\partial T}{\partial \lambda_2} = 2\lambda_1 S[(x_1-\bar{x}_1)(x_2-\bar{x}_2)] + 2\lambda_2 S(x_2-\bar{x}_2)^2$

$\frac{\partial D}{\partial \lambda_1} = d_1$, $\frac{\partial D}{\partial \lambda_2} = d_2$

Thus the equations of estimation become

$$\lambda_1 S(x_1-\bar{x}_1)^2 + \lambda_2 S[(x_1-\bar{x}_1)(x_2-\bar{x}_2)] = d_1$$

$$\lambda_1 S[(x_1-\bar{x}_1)(x_2-\bar{x}_2)] + \lambda_2 S(x_2-\bar{x}_2)^2 = d_2$$

which are of the familiar multiple regression type and may be solved in the same way as multiple regression equations. Putting 1 and 0, and 0 and 1 respectively for d_1 and d_2 gives two pairs of equations whose solutions are

c_{11} c_{12}

c_{21} c_{22}

from which $\lambda_1 = c_{11}d_1 + c_{21}d_2$ and $\lambda_2 = c_{12}d_1 + c_{22}d_2$

Lederberg

1.

107	30	137
76	6	82
183	36	219

a_1	a_2	a_1+a_2
a_3	a_4	a_3+a_4
a_1+a_3	a_2+a_4	n

$$\chi^2_{[1]} = \frac{(a_1 a_4 - a_2 a_3)^2 n}{(a_1+a_2)(a_3+a_4)(a_1+a_3)(a_2+a_4)}$$

$$= \frac{(642 - 2,280)^2 \times 219}{137 \times 82 \times 183 \times 36} = \frac{587,586,636}{74,009,592}$$

$$= 7.9393$$

2.

133	53	186
80	23	103
213	76	289

$$\chi^2_{[1]} = \frac{(3,059 - 4,240)^2 \times 289}{186 \times 103 \times 213 \times 76} = \frac{403,085,929}{310,129,704}$$

$$= 1.2997$$

3.

19	5	24
49	8	57
68	13	81

$$\chi^2_{[1]} = \frac{(152 - 245)^2 \times 81}{24 \times 57 \times 68 \times 13} = \frac{700,569}{1,209,312}$$

$$= 0.5793$$

Sederberg

		$+B_1$	B_1	n	χ^2	$\frac{-B_1^2}{n}$
4.	$-r$	100	30	130	0.2117	6.9231
	$-s$	50	21	71	0.8611	6.2113
	$+r$	55	17	72	0.0564	4.0139
	$+s$	4	1	5	0.0623	0.2000
	Tot.	209	69	278	1.1915	17.1269

For $-r$ $\quad \chi^2_{[1]} = \frac{(100\times69)-(30\times209)^2}{209\times69\times130} = \frac{396{,}900}{11{,}874{,}730} = 0.2117$

For $-s$ $\quad \chi^2_{[1]} = \frac{(50\times69)-(21\times209)^2}{209\times69\times71} = \frac{881{,}721}{1{,}023{,}891} = 0.8611$

For $+r$ $\quad \chi^2_{[1]} = \frac{(55\times69)-(17\times209)^2}{209\times69\times72} = \frac{58{,}564}{1{,}038{,}312} = 0.05642$

For $+s$ $\quad \chi^2_{[1]} = \frac{(4\times69)-(1\times209)^2}{209\times69\times5} = \frac{4{,}489}{72{,}105} = 0.0628$

B&S

	O	R	
4	202	76	278
2	213	76	289
	415	152	567

$$\chi^2_{[1]} = \frac{(15{,}352-16{,}188)^2\times567}{278\times289\times415\times152} = \frac{396{,}274{,}032}{5{,}067{,}973{,}360} = 0.0782$$

	O	R	
1&3	251	49	300
2&4	415	152	567
	666	181	867

A $\quad \chi^2_{[1]} = \frac{(38{,}152-20{,}335)^2\times867}{300\times567\times666\times181} = \frac{275{,}225{,}288{,}963}{20{,}504{,}874{,}600} = 13.4224$

	O	R	
1&2	396	112	508
3&4	270	89	359
	666	201	867

$$\chi^2_{[1]} = \frac{(35{,}244 - 30{,}240)^2 \times 867}{508 \times 359 \times 666 \times 201} = \frac{21{,}709{,}693{,}872}{24{,}413{,}410{,}152} = 0.8893$$

	O	R	~~Total~~ n	%R	χ^2	$\frac{R^2}{n}$
1	183	36	219	16.44	5.5947	5.9178
2	213	76	289	26.30	1.5738	19.9862
3	68	13	81	16.05	0.3148	2.0864
4	202	76	278	27.34	2.6946	20.7770
Total	666	201	867	23.18	12.1779	46.5986

For 1 $\quad \chi^2_{[1]} = \frac{(183 \times 201) - (36 \times 666)^2}{666 \times 201 \times 219} = \frac{164{,}019{,}249}{29{,}316{,}654} = 5.5947$

2 $\quad \chi^2_{[1]} = \frac{(213 \times 201) - (76 \times 666)^2}{666 \times 201 \times 289} = \frac{60{,}886{,}809}{38{,}687{,}274} = 1.5738$

3 $\quad \chi^2_{[1]} = \frac{(68 \times 201) - (13 \times 666)^2}{666 \times 201 \times 81} = \frac{25{,}100{,}100}{10{,}843{,}146} = 0.3148$

4 $\quad \chi^2_{[1]} = \frac{(202 \times 201) - (76 \times 666)^2}{666 \times 201 \times 278} = \frac{100{,}280{,}196}{37{,}214{,}748} = 2.6946$

Lederberg

4 $S\left(\frac{-B_1^2}{n}\right) - \frac{Tot.\,-B_1^2}{Tot.\,n} = 0.2224$

$\frac{Tot.\,n^2}{Tot.\,+B_1 \times Tot.\,-B_1} = \frac{278^2}{209 \times 69} = \frac{77284}{14421}$

$= 5.3591$

$\chi^2_{(3)} = 0.2224 \times 5.3691$
$= 1.1919$

BrS

$S\left(\frac{R^2}{n}\right) - \frac{Tot.\,R^2}{Tot.\,n} = 2.1688$

$\frac{Tot.\,n^2}{Tot.\,O \times Tot.\,R} = \frac{867^2}{666 \times 201} = \frac{751689}{133866}$

$= 5.6152$

$\chi^2_{[3]} = 2.1688 \times 5.6152$
$= 12.1782$

Lederberg — 2x2 method

1.

	S	r	
c	107	30	137
r	76	6	82
	183	36	219

a_1	a_2	a_1+a_2
a_3	a_4	a_3+a_4
a_1+a_3	a_2+a_4	n

$$\chi^2 = \frac{(a_1a_4 - a_2a_3)^2\, n}{(a_1+a_2)(a_3+a_4)(a_1+a_3)(a_2+a_4)}$$

2.

c	133	53	186
r	80	23	103
	213	76	289

3.

	S	r	
c	19	5	24
r	49	8	57
	68	13	81

	O	R	Total
1.	183	36	219
2.	213	76	289
3.	68	13	81
	464	125	589

2x2 method

(1+3)

183	36	219
68	13	81
251	49	300

(1+3) & 2

251	49	300
213	76	289
464	125	589

Brandt's Snedecor's method

Lederberg

4

	$+B_1$	$-B_1$	
$-r$	100	30	130
$-s$	50	21	71
$+r$	55	17	72
$+s$	4	1	5
	209	69	278

B/S

2x2

	D	R	
4	202	76	278
2	213	76	289
	415	152	567

	O	R	Total
1	183	36	219
2	213	76	289
3	68	13	81
4	202	76	278
	666	201	867

% R

	O	R	
193	251	49	300
294	415	152	567
	666	201	867

	O	R	
142	396	112	508
394	270	89	359
	666	201	867

B/S