Algorithm for
Finding a Hamilton Circuit of a Cyclic Graph

A Hamilton circuit is a continuous closed path, through some of the edges of a cyclic graph, that traverses every node just once. Hamilton's own examples are the dodecahedron and icosahedron, which can be represented as planar nets, the circuit being marked, as follows:

linear programming and graph isomorphism

~~When~~ a Hamilton circuit (HC) when they can be found, can be very useful in solving network problems which arise in connection with & in the present instance, the impetus for an efficient algorithm to find an HC came from the systematics of chemical graphs, i.e., of the molecular structure of organic compounds. An incidental problem was the search for and verify the smallest, of the trihedral polyhedra lacking an HC (~~[illegible]~~ which ~~by~~ Tait conjectured to be nonexistent; but Tutte displayed an example of order 46).

180 times

of ways of accessing every node of the cut graph. To form a Hamilton circuit, two cut graphs must be united, edge by edge, which satisfy ~~their~~ the requirement that their lane sets are mutually compatible.

~~to form~~ At each stage of the tree, the lane set is ~~[illegible]~~ first expanded, each node bringing pendant 2 cut edges, either of which is at this point equally available as a lane. Then the cut edges are scanned for ~~redundancy equivalence with one another; such~~ identities which edges can now be joined and thereby removed from the set of cut edges. At this time, the lane-set is diminished with respect to items involving the reunited edge. If the lane-set is ~~still~~ void

~~Much of our~~ The discussion will be illustrated mainly with such polyhedra regarded as undirected graphs or 3-nets. In a fairly obvious way, however, the argument can be extended to cyclic graphs generally.

These graphs are ~~equal~~ trivalent at each node and everywhere at least 3-connected. That is, the 3-net cannot be separated by fewer than 3 cuts. It is also planar, i.e., can be represented on the plane (Schlegel diagrams) or on the ~~two~~ 2-sphere (convex polyhedra) with no crossing edges.

n stands for the order (number of nodes) of a graph or cutgraph

q for the level of cut edges

p for the connectedness in a special sense: the least number of cuts needed to extract a cutgraph larger than a single node or node-pair. Evidently $3 \leq p \leq 5$. Recall also Euler's result that a polyhedron must have at least one face, $3 \leq n \leq 5$.

g_i stands for a graph drawn in Figure ___.

[N] stands for the incidence matrix (connection table) of the graph, showing which nodes adjoin which

[E] is the corresponding matrix for edges.

The only problem attacked here is a combinatorial one, that can rapidly evade a brute force approach even by a fast computer. Consider two approaches.

(a) $n!$ Apply the symmetric group S_n to the labels of the nodes. Test each permutation for the serial adjacency of the nodes. This approach is prohibited well before $n \geq 14$

(b) 2^n A well-known method (Berge*) applies a binary search. From n_1 a binary choice is made between two* of its adjacent nodes, and this process iterated until ~~a HC is found, or~~ the path closes. If this path encompasses n nodes (hence n choices) an HC has been found. Then the ~~alternative choice is made of the~~ path is retraced to the last option point and the alternative choice selected. All possible paths can be described with n bits, hence the ~~max~~ upper limit is 2^n choices. However, many searches trees will be truncated quite early. In practice, about $2^n/n^2$ choices have sufficed to find one HC when it exists.

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* choosing the third node will trace paths through the others that must be reflections of paths. Some reflections will already be included

This method is feasible and quite reasonable through $n \geq 20$, ~~and~~ trivial through $n \leq 12$. It has been used for a complete cycheap testing of the HCs of most of the cyclic graphs relevant to organic chemistry, i.e., all trivalent graphs, $n \leq 12$, and many 3-nets $n \leq 20$.

For recurrent applications at $n > 12$ binary search is still troublesome. Furthermore it opens up little insight into the conditions for the existence of an HC in some larger 3-net, say $n > 30$. Note that Tutte's example is n_{46}, although an example has been found (independently also by and by) at n_{38}.

A tree-building method. The net can be searched (or built) ~~in the fashion of~~ as a tree, starting from an arbitrary node. Each branch is a cut edge, obtained by planting a node and its pendant edges on one of the existing cut edges, ~~This process is governed by~~ according to [N] and [E].

We now define a lane-set and the lane-set list. A lane-set is ~~a pair, or set of pairs~~ one or more pairs of cut edges ~~which might be~~ are possibly available for Hamilton circuits; ~~if~~ [illegible] it is the set