THE USE OF A LOGARITHMIC AMPLIFIER
IN
DATA PROCESSING OF ANALOG SIGNALS

Walter E. Reynolds

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Prepared by Walter Reynolds

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Instrumentation Research Laboratory, Department of Genetics
STANFORD UNIVERSITY SCHOOL OF MEDICINE
Palo Alto, California

Abstract

A number of aspects of usage of a logarithmic amplifier for data compression and low level resolution are given. These include an example of data interpretation and a derivation of error figures. An appendix contains a circuit and specifications of the logarithmic amplifier used.

Contents

Illustrations

I. Introduction.

Outputs of some physical and biological detection instruments contain information over a very wide range of values. In the case of a mass spectrometer this range can exceed 1 : 30,000. Visual interpretation of analog signals has a normal resolution of only 1 : 100 to 1 : 1000. Analog to digital conversions may be used, but also seldom exceed 1 : 1000. In cases where visual interpretation is made from strip chart recordings, some improvement is obtained by multiple trace recordings, each trace at a different gain so that at least one of the traces has a usable range in all points of interest. Similar digital techniques, however, are difficult and awkward.

A logarithmic data transformation can make a very useful data compression device. Such techniques have been employed here with some success and much promise. An amplifier based upon a circuit reported by J.F. Gibbons and H.S. Horn[1] has been built by the Genetics Department Instrumentation Research Laboratory, Stanford Medical School, and used there and in the Chemistry Department, Stanford University (See Addendum to this report detailing a Model 100-9 Logarithmic Amplifier.) Particular applications have included analog to digital recording of mass spectrometer data and logarithmic analog to strip chart recordings of similar data to detect very low level metastable ion phenomena. A forthcoming communication in Analytical Chemistry[2] describes the uses and advantages of logarithmic data transfer to the identification of metastable ions. In this case the useful information was at an amplitude of only 1 part in 100,000 of the peak amplitudes.

> This (logarithmic) system has been used in the author's
> laboratory in conjunction with an Atlas CH-4 mass spec-
> trometer, the recording system of which suffers from the
> disadvantage that it requires manual attenuation in the
> recording of a given mass spectrum, necessitating several
> scans if one wishes to obtain accurate intensities of
> both the strong and weak peaks. This is especially true
> if one is trying to observe metastable ions which appear
> as weak broad peaks. [3]

Further, it was noted that the logarithmic plots show a considerable number of metastable peaks not apparent in linear recordings, but also produce a spectrum which is far easier to count. The logarithmic plot was

able to record all the information in one scan.

Logarithmic data transfer promises to be very useful as a prior step in analog to digital conversions for digital computer analysis of mass spectra and other similar signals. Analog to digital conversion effectively wipes out all linear low level signals that lie below the minimum analog to digital resolution. An improvement in resolution by a factor of as much 10^6 can be obtained by means of the logarithmic transformation.

II   Effect of Linear Error on Transmission of Logarithmic Data.

The errors resulting from noisy handling of logarithmic analog signals are quite different from those produced by normal linear analog data processing. In this section the error expressions are derived from retrieved data. It will be shown that for large signals the error ratio is increased and that for small signals it is decreased. In all cases the error is proportional to the signal.

The logarithmic analog signal will in almost all cases be handled by some linear data transmission device. This may be a strip chart recorder, magnetic tape, meter movement, analog to digital converter or some combination of these. It is typical of this class that they have an uncertainty or error that is a constant percentage of the maximum signal processible.

The following model is assumed. An ideal logarithmic function is assumed for the amplifier and an ideal antilogarithmic output function is assumed at the output. All errors are assigned to the data transmission device.

Logarithmic amplifier
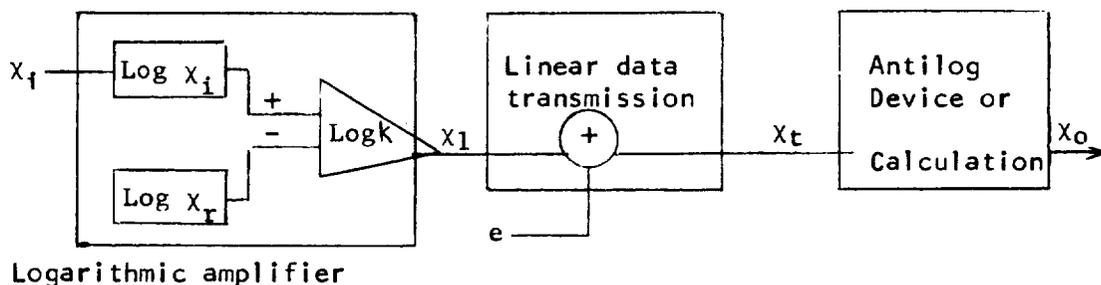
Figure 1.   An assumed model of data transmission.

2.

Logarithms are defined only for positive numbers; hence the log amplifier has an output for only positive values. Furthermore, the output of the model 100-9 log amplifier has a minimum output corresponding to roughly 1, or $x_i = x_r$. And there is a maximum limit of x, x_{max}, imposed by physical limitations of the logging amplifier. Hence the log amplifier may be used over a range of $x_r < x_i < x_{max}$. It should also be noted that the user may elect to scale his input to some x_{max} less than that of the amplifier.

The log amplifier has an output

$$x_1 = \log_b k(x_i/x_r)$$

where $\log_b k$ is a scale factor of the log amplifier. If $k = x_r$

$$x_1 = \log_b x_i$$

If $k \neq x_r$ it only applies a scale multiplier to the output and does not enter into the error analysis.

Let the dynamic range used be R, defined as $R = x_{max}/x_r$.

The output of the log amplifier will range from $\log_b kx_r$ to $\log_b kx_{max}$.

$$x_1 \max - x_1 \min = \log_b kx_r - \log_b kx_{max}$$

$$= \log_b R$$

Note that b is undefined. The output curve of the log amplifier could represent $\log x_i$ to any base. Selecting the base simply puts analog numerical values at points on the curve. R may be expressed as a ratio, decades, nepers, or octaves. For example, with $R = 10^9$, R is 9 decades, 20.7 nepers, or 29.9 octaves. Similarily $\log x_i$ may be expressed or converted to any of these quantities.

It is normal in linear data transmission devices to express noise as a fraction of the maximum signal. Following this custom and assuming that $\log_b R$ is scaled to be equal to this maximum signal,

$$e = N \log_b R$$

where N is the noise ratio of the linear data transmission device.

The output of the linear data transmission device has an output of:

$$x_t = \log_b x_i + e \qquad\qquad (\text{assumed } k = x_r)$$

$$= \log_b x_i + \log_b R^N$$

$$= \log_b x_i R^N$$

After taking antilog to base b through the antilogarithmic device,

$$x_o = x_i R^N$$

If N is small compared to R,

$$x_o = x_i \left(1 + N \ln R \right)$$

where ln is the natural logarithm.

Letting $x_o = x_i + e_o$ where e_o is error at the output,

$$x_i + e_o = x_i + x_i \, N \ln R$$

$$e_o = x_i \, N \ln R$$

It can be seen that the uncertainty (error) after data retrieval by the antilog operation has a value equal to a constant ratio of the input signal, x_i. And this ratio is the noise factor of the data transmission device times the natural logarithm of the allowed range of x .

An interesting value to investigate is that value for which the errors of a linear and logarithmic signal are equal. This would be the value of x_i which would produce equal uncertainty whether transmitted in linear or logarithmic form:

$$e_o = e$$

$$x_i \, N_1 \, \log R = N_2 \, x_{max}$$

where N_1 is the noise factor during the logarithmic transmission and N_2 is the noise factor during linear transmission. If indeed $\log R = x_{max}$ in the two cases, then $N_1 = N_2$.

$$\frac{x_i}{x_{max}} = \frac{1}{\ln R} \frac{N_2}{N_1}$$

4.

If R is in decades ln R is approximately 2.3 R.

$$x_1/x_{max} = (1/2.3R) \; (N_2/N_1)$$

Therefore, for a value of x_1 below that given by the above equation, the accuracy of the retrieved values of x_1 is improved by logarithmic processing, and for values above that the accuracy decreases.

The following example illustrates these points: the linear transmitting device is a strip recorder with an accuracy given as 0.2% of full scale, 5 inches. x_r was selected as 1 mv and set at 1 inch.

N_1 calculated as a ratio over the 4 inches used is

$$N_1 = .002 \; (5/4)$$

$$= .0025$$

R over this same range is 4 decades $= 10^4$. ln r = 9.2

x_{max} then is 1 mV x 10^4 = 10 volts

Error ratio of retrieved data is then

$$N_1 \; \ln R = .0025 \; x \; 9.2 \qquad \text{or } 2.3\% \text{ at any amplitude.}$$

The point of equivalent error as compared with linear chart recording can be found. If x_{max} of the linear recording is 5 volts, then

$$N_2 = .001 \quad \text{referred to 10 inches full scale (10 volts)}$$

$$x_1 = \frac{1}{9.2} \; x \; \frac{.002}{.0025} \; x \; 10 = .44 \text{ volt}$$

Figures 2 and 3 illustrate what may be expected. Figure 2 is a linear recording of an event. At the right are plotted values of expected uncertainty. Figure 3 is the recording on the same recorder of a logarithmic signal with, again, the expected uncertainty. By comparing Figure 2 with Figure 3 it can be seen that e_0 has the same value on both plots at .44 volts. Also e_0/x_1 has the same value at .44 volt. At signal points above this voltage the linear recording can be expected to have greater accuracy. For signal points lower than .44 volts greater accuracy can be expected with the logarithmic processing.
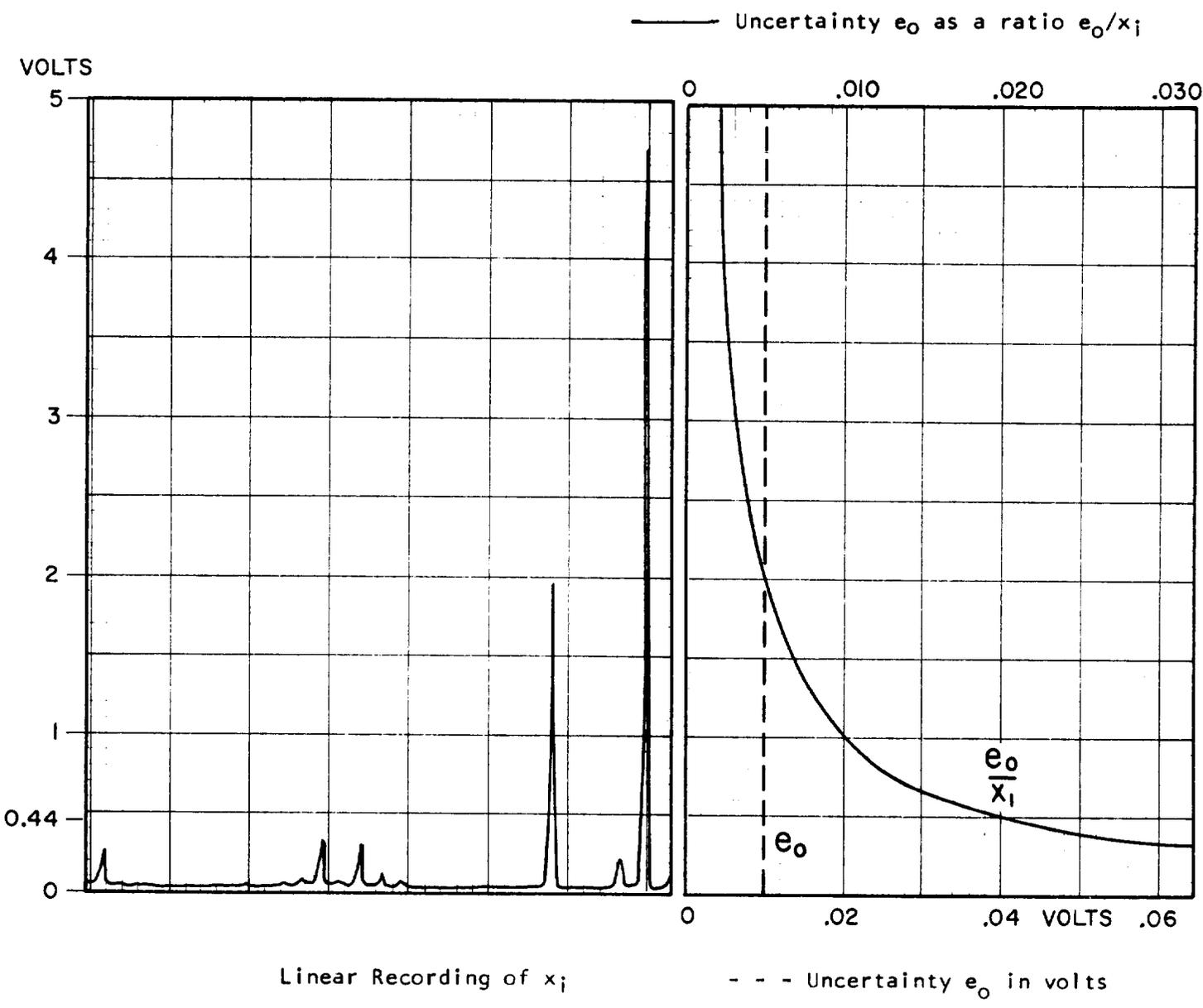
Figure 2. A linear recording of an event. Recorded on a 5" chart with an accuracy of 0.2% full scale. The curves at the right plot this uncertainty as a function of the signal.
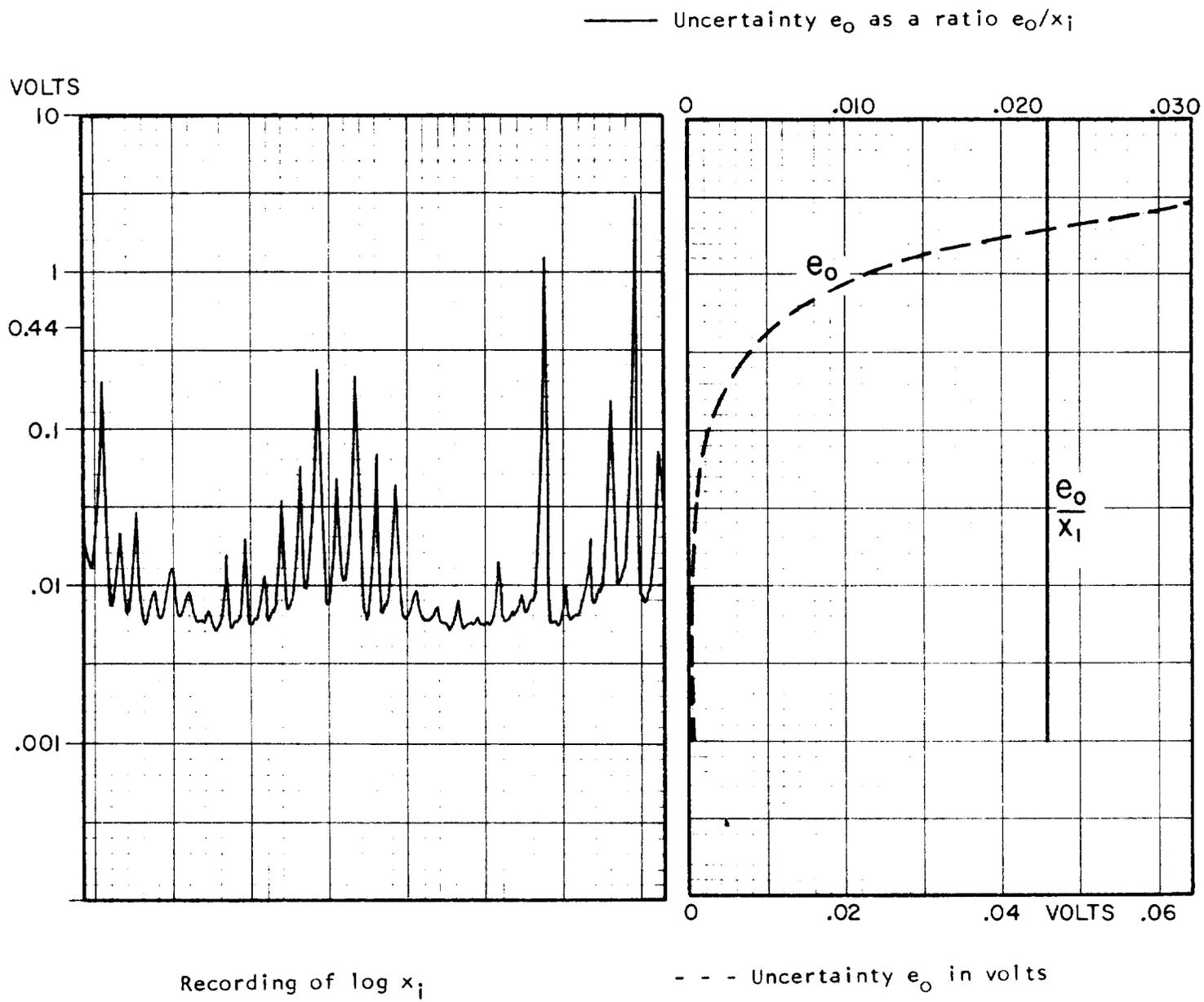
Figure 3. A logarithmic recording of a similar event to that of Figure 1. This is the recording of log x_i. The curves at the right plot uncertainty as a function of the logarithmetic signal amplitude.

(Note: Figure 2 and 3 are of similar, but not identical, signals.)

III   An Illustrative Example of Data Interpretation.

Figure 4 is a reproduction of a logarithmic plot from a mass spec-
trometer. This has much usable information, but is also a severe ex-
ample of an effect a logarithmic amplifier will accentuate: "drifting
base line" in the signal. The very high resolving power of the logar-
ithmic amplifier at very low signal levels will make what was an other-
wise acceptable zero drift appear to be a major portion of the signal.
However, upon careful examination it will be seen that no information
is actually lost.

The "drifting base line" deserves some special explanation and
evaluation. The signal of Figure 4 is from an amplifier on a mass spec-
trometer that has a 0 to 20 V dc output range. The amplitude of the
signal between well defined peaks is theoretically zero, but some drift
is inevitable. Only the amount of drift is subject to criticism or per-
haps engineering improvement. Since this signal was normally recorded
in a linear manner at about 0 to 5 V range, a drift of 0.1%, or 5 mV,
would probably be acceptable. The instrument would be adjusted so that
this drift was plus or minus about zero.

If this signal is to be processed through a logarithmic amplifier,
other considerations must be made. The log of a negative number is un-
defined and the log of zero is minus infinity. Both lie out of the
range of physical realization by an amplifier. The logarithmic ampli-
fier used has a number of reference settings, from 1 μV to 10 mV. When
set at a particular reference, only signals between that reference and
the upper input limit of the amplifier give meaningful output. Signals
below the reference are lost in negative saturation (actually a toler-
ance of about .5 to .7 of the reference is permitted, giving an output
of about .8 as a minimum.)

To insure that the signal to the logarithmic amplifier is never nega-
tive, it is recommended that the zero set of the signal be offset in the

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positive direction so that the lower excursions of the signal remain within the allowable limits of the logarithmic amplifier; i.e., above the reference selected. Since this can be as low as 1 μV, this "offset positive" can be almost arbitrarily low. In fact it turns out that the logarithmic amplifier is by far more sensitive to detection near zero than any other detector used normally on such systems, and hence the "offset positive" can be nearer zero than supposed zero settings using linear detectors.

The Model 100-9 logarithmic amplifier has "Reference" and "Scale" settings; with these, an output corresponding to log (signal/reference = 1) and log (signal/reference = 10^n) may be obtained. In Figure 4, the recorder was adjusted so that reference (1 mV) was set at 1 inch; log 10, or one decade (10 mV) was set at 2 inches; two decades at 3 inches, etc.

Conveniently, this has now scaled the chart so that the inches, expressed in decimal form $\pm$ from the first inch level, are the logarithm to the base 10 of the amplitude divided by the reference. The "zero drift" can be seen to begin at about 1 mV ($\log_{10}^{-10}$ 0.65) = 4.4 mV. This drifts downward until it passes below the reference and then rises to about 10 mV.

Examples of calculation of peak amplitude (all values expressed in mV):

Peak amplitude of "A" = antilog 1.58 - antilog .55

$$= 38 \text{ (peak)} - 3.5 \text{ (zero offset)}$$

$$= 34.5$$

Others are:

"B" = antilog 2.29 - antilog .48 = 195 - 3 = 192

"C" = antilog 3.57 - antilog .70 = 3720 - 5 = 3715

"D" = antilog .84 - antilog .0 = 6.9 - 1 = 5.9

"E" = antilog .94 - ? = 8.7 ? = 8.7
    Information about zero line is lost.

"F" = antilog 3.28 - antilog 1.0 = 1910 - 10 = 1900
    The local rise in base line here appears to be lack of resolution between peaks within the spectrometer.

"G" = antilog 1.23 - antilog 1.02 = 17.0 - 10.5 = 6.5
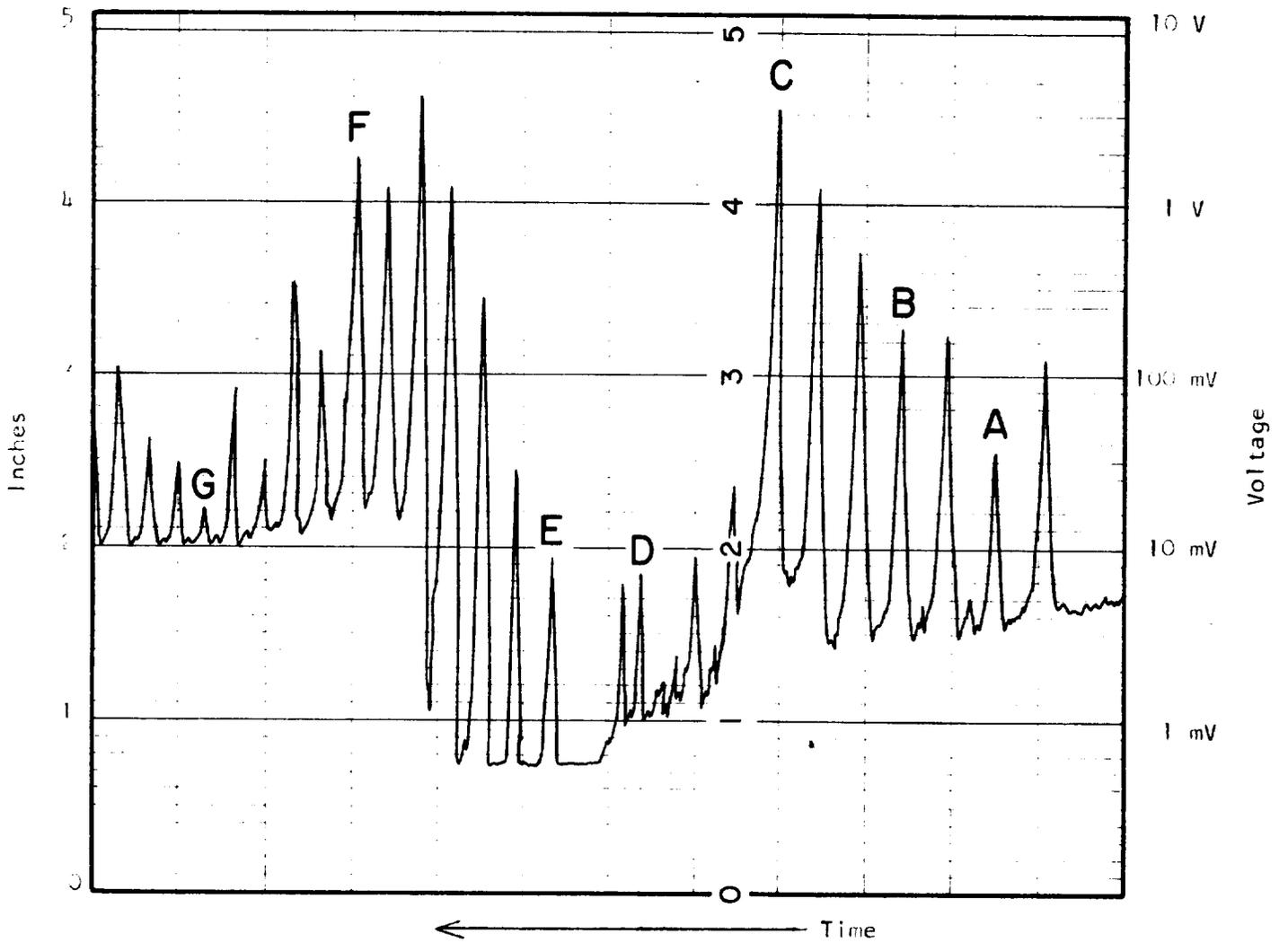
Figure 4.   An example of a recorded spectrum.

It was shown in Section II that the uncertainty due to errors in the data transmission would be a constant percentage of the signal. In this case the recording and reading back of the signal on a strip chart recorder would be the data transmission. Assuming a .5% accuracy of full scale inches of the chart, $N = .005$. The range is five decades or 10^5; $\ln 10^5 \simeq 11.5$. The uncertainty is then $n \ln R = 5.75\%$.

A summary of the above values with expected uncertainty versus the expected uncertainty in linear recording is informative:

Peak	Amplitude mV	Expected uncertainty with log transmission		Expected uncertainty with linear transmission	
		mV	percent	mV	percent
A	34.5	2.0	5.8 %	25	73.0 %
B	192.0	11.0	5.8 %	25	13.0 %
C	3715.0	212.0	5.8 %	25	.7 %
D	5.9	0.3	5.8 %	25	425.0 %
E	8.7	0.5	5.8 %	25	285.0 %
F	1900.0	108.0	5.8 %	25	1.3 %
G	6.5	0.4	5.8 %	25	390.0 %

IV   Further Work Indicated·

The signal of Figure 4 was an example of very severe base line drift, and is not supposed to be an example of a desirable signal; however, the problem does exist to some extent with any real analog signal. In general it may be said that use of the logarithmic amplifier with signals that have base line drift does not result in the loss of any information, but base line drift does limit the usefulness of the logarithmic amplifier to resolve very small signals.

Further work is being done to investigate the possibility of using the logarithmic amplifier itself to detect base line drift and to generate an appropriate error signal. Then this error signal could control a dc bias applied to the original signal to drive the base line to a preselected reference.

V   Sources of Logarithmic Transfer Elements.

The amplifier described in Appendix A uses a Fairchild FSP-30 PNP silicon transistor as the logarithmic element. This is employed as described in reference (1).

Other commercial elements are available. Nexus Research Laboratory, Inc., Canton, Massachusetts has a Type LGR-6 Logarithmic Ratio Module. George A. Philbrick Researches, Inc., Boston, Massachusetts, has a type PL1 module and others under development.

Principle manufacturers of oscilloscopes and x-y recording equipment are known to be developing instruments with electronic logarithmic conversions. It is expected that such instruments, with visual logarithmic displays, will soon be announced as commercially available.

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3. Ibid.

4. Lederberg, J.; An Instrumentation Crisis in Biology. NASA Status Report through April 1, 1963: "Cytochemical Studies of Planetary Microorganisms, Explorations in Exobiology"